Numerical Development

Daniel C. Hyde
Psychology 216

Why study number?

- Drives scientific discovery and technological advancement
- Used daily to buy things, count things, etc.
- Taught in nearly all levels of education
  - Improve math education
    - U.S. eighth graders rank 9th in math and 11th in science out of 47 countries (Gonzales et al., 2009)
  - Treat dyscalculia
    - difficulty learning/comprehending mathematics
      - 1%-7% of the population (P.O.S.T., 2004)

Where do these abilities come from?

- Uniquely human invention?
- Based in more primitive quantitative abilities?

Number as we know it

- When we think of number we typically think of the positive integers or NATURAL NUMBER
  - Numbers we use to count
  - Number words we know (seven, ten, etc.)
  - Numbers we use in arithmetic and other math operations
- Properties of the integer system
  - Each number represents an exact cardinal value
  - Each number has a unique successor
  - Each number (n) can be succeeded by n+1

How do we acquire these concepts?

Piaget's argument

- Numbers are a cultural construction
  - Cannot be represented in terms of those sensorimotor primitives we have at birth
- Preschool children lack certain logical capacities necessary to acquire number concepts
  - the capacity to represent serial order, the capacity to represent the inclusion relations among sets.
- Must acquire these capacities via sensorimotor experience before number can be mastered.
- Not until age 5 or 6 that children construct a complete concept of number (positive integers).

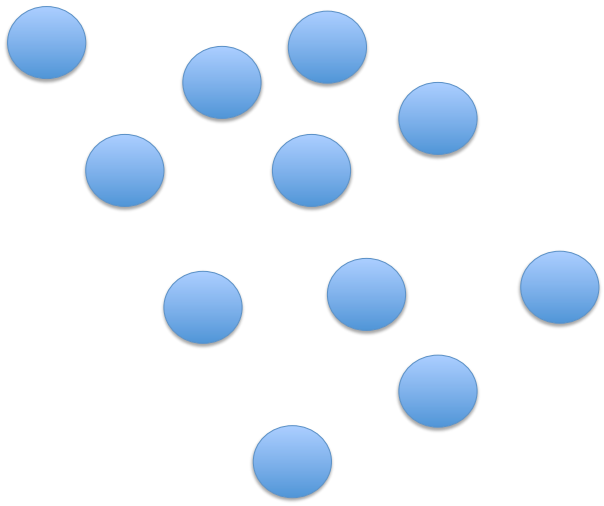
Piaget's Conservation of Number

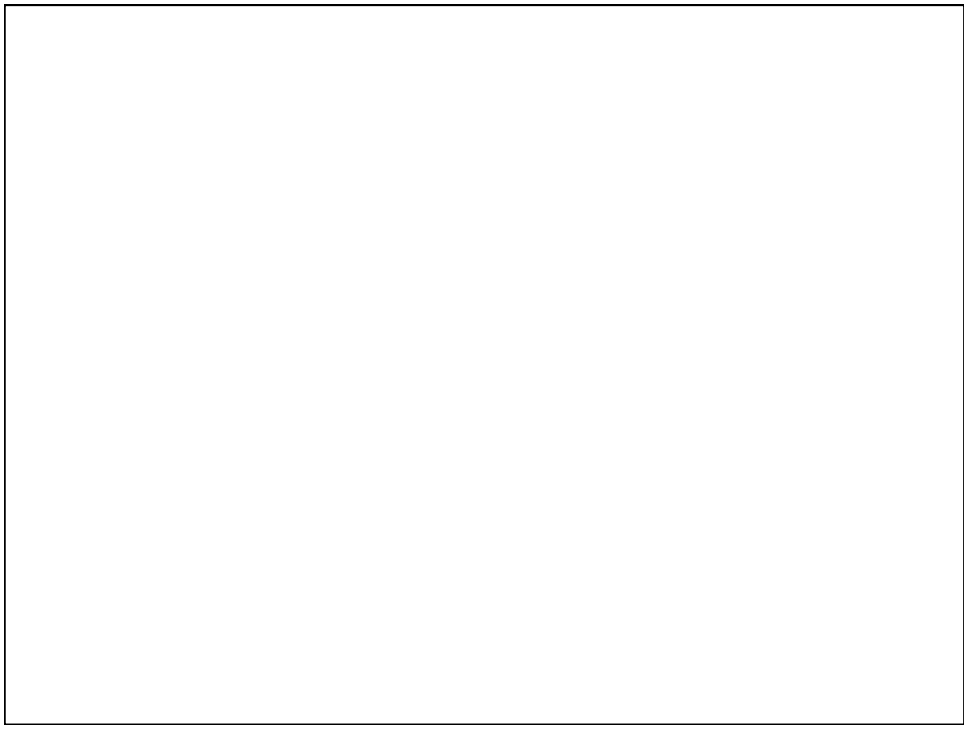
Overview of numerical development lecture

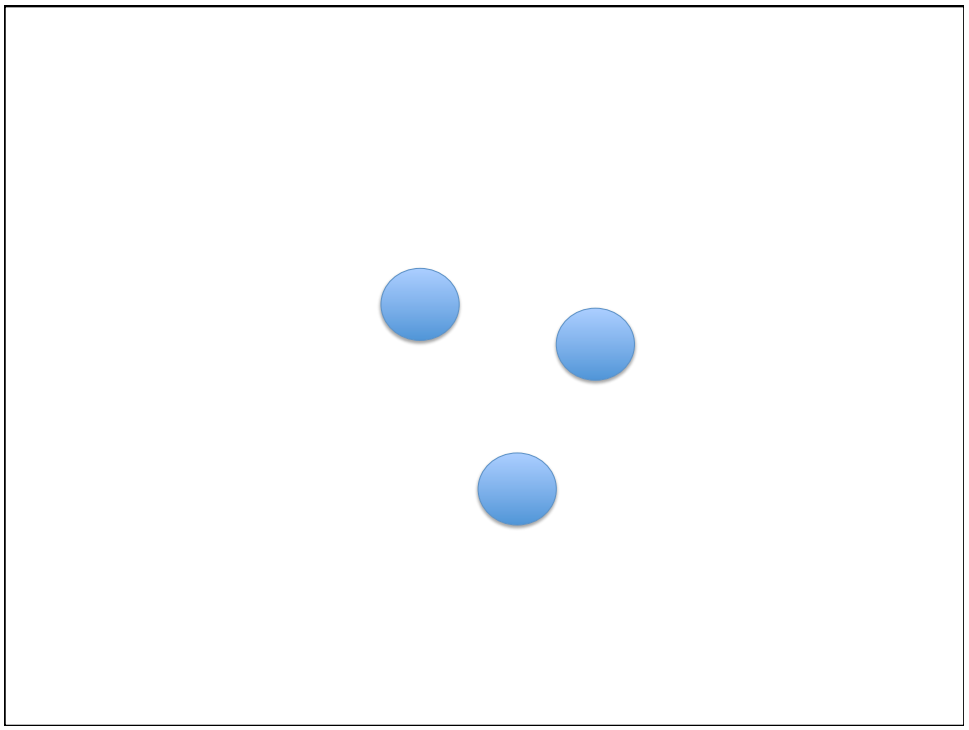
- Nature of innate number concepts
  - What are the two core number systems?
  - In what sense are they numerical?
- Learning natural number/integer concepts
  - What are the stages of counting development?
  - What is the role of core number in counting development?
- Core number concepts and math education
  - What is the relationship between the core number systems and math achievement/development?

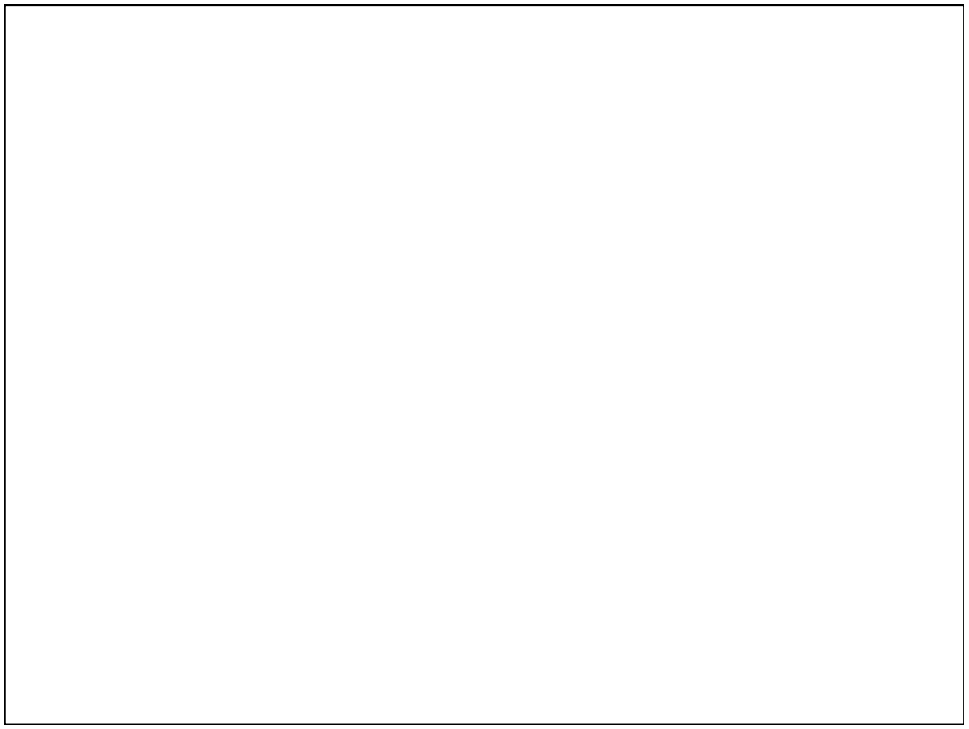
What numerical abilities do we have before passing the conservation task?

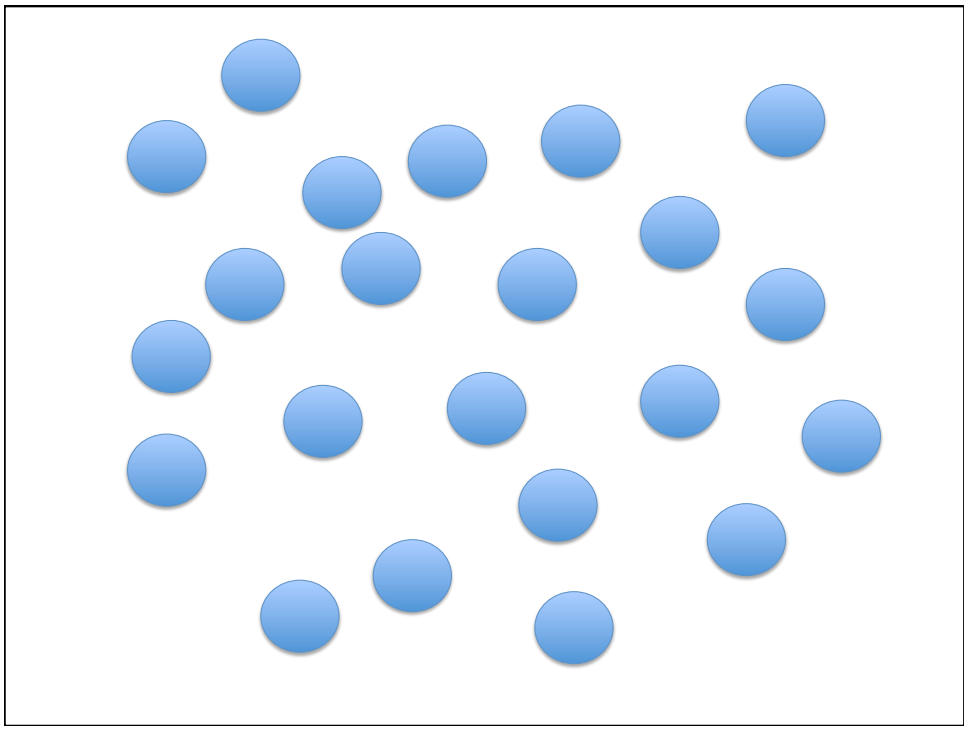
- Two non-verbal, non-symbolic systems
  - Approximate Number System (ANS)
    - "Number Sense"
    - Ability to approximate cardinal value of set

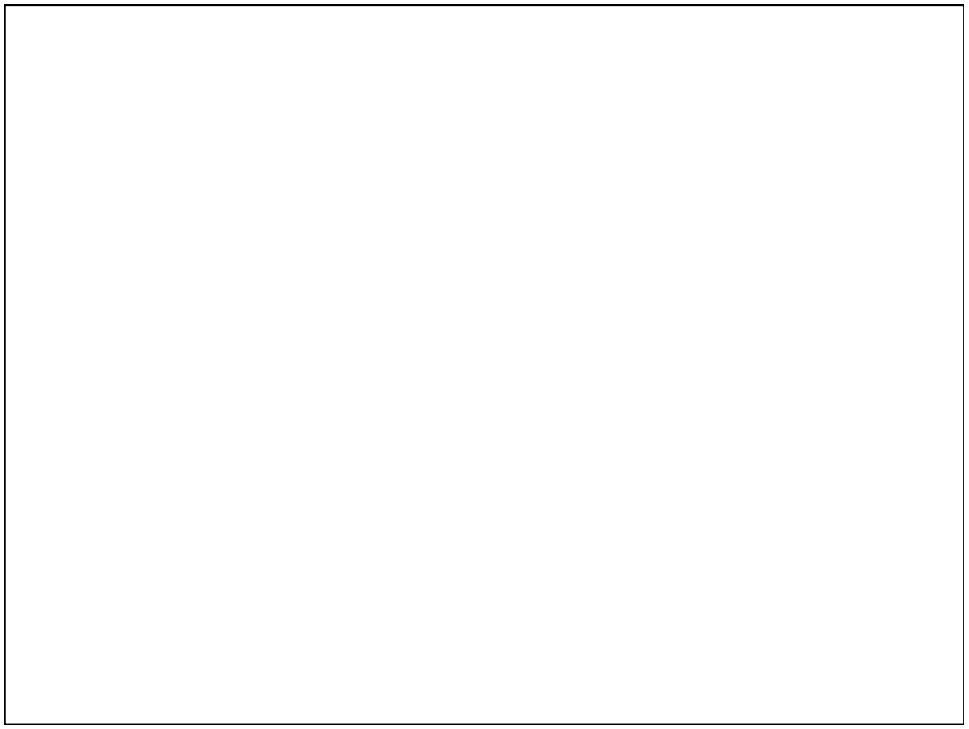


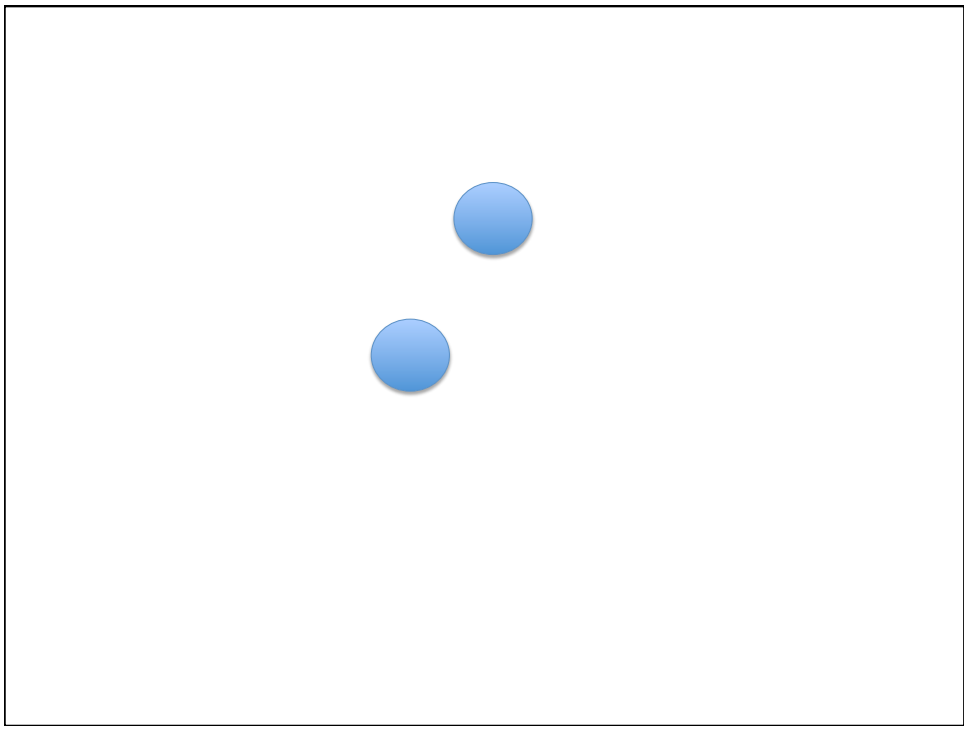




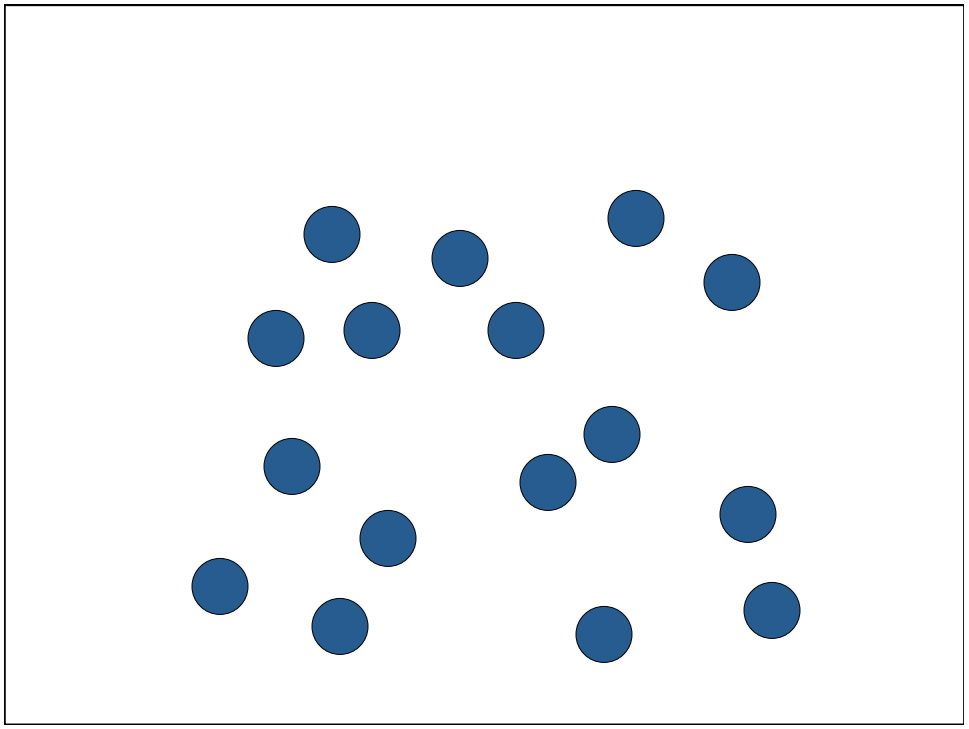


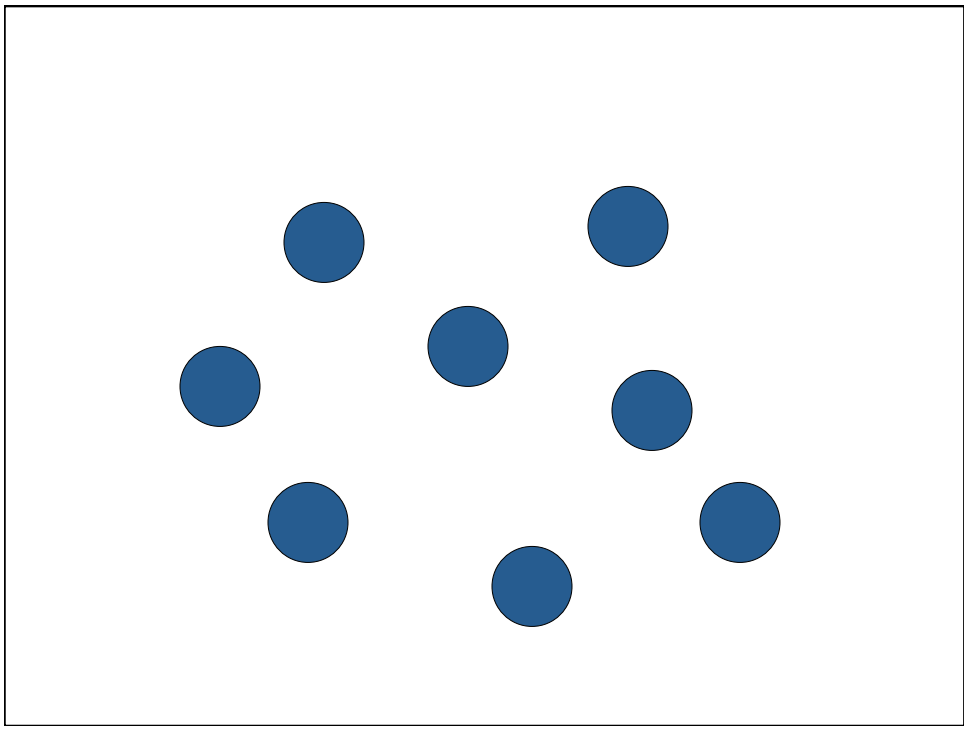




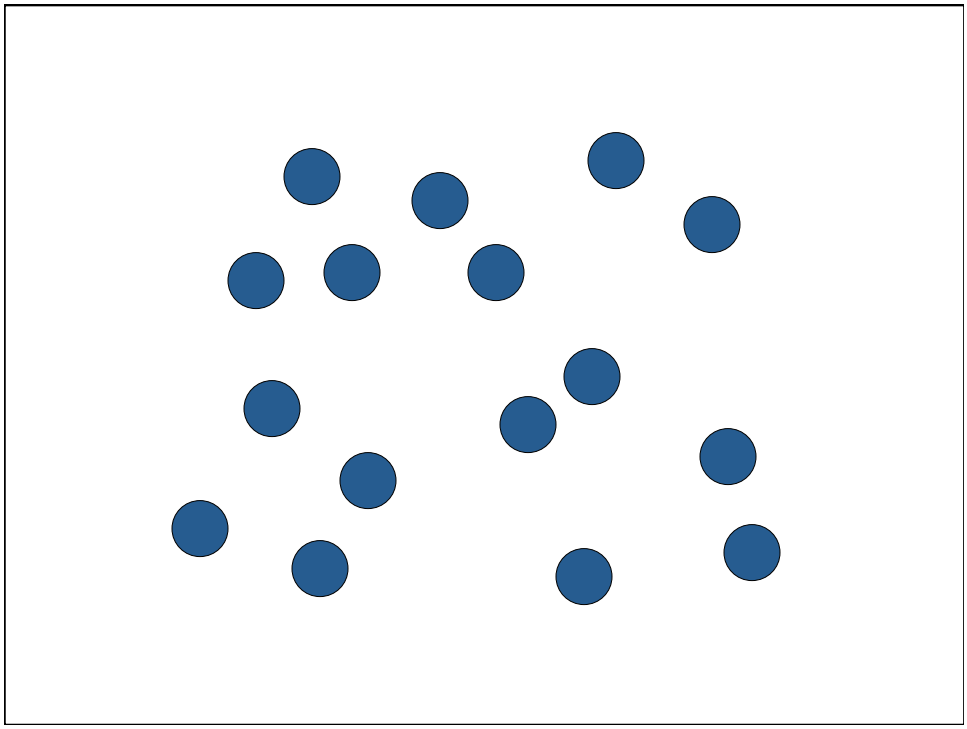


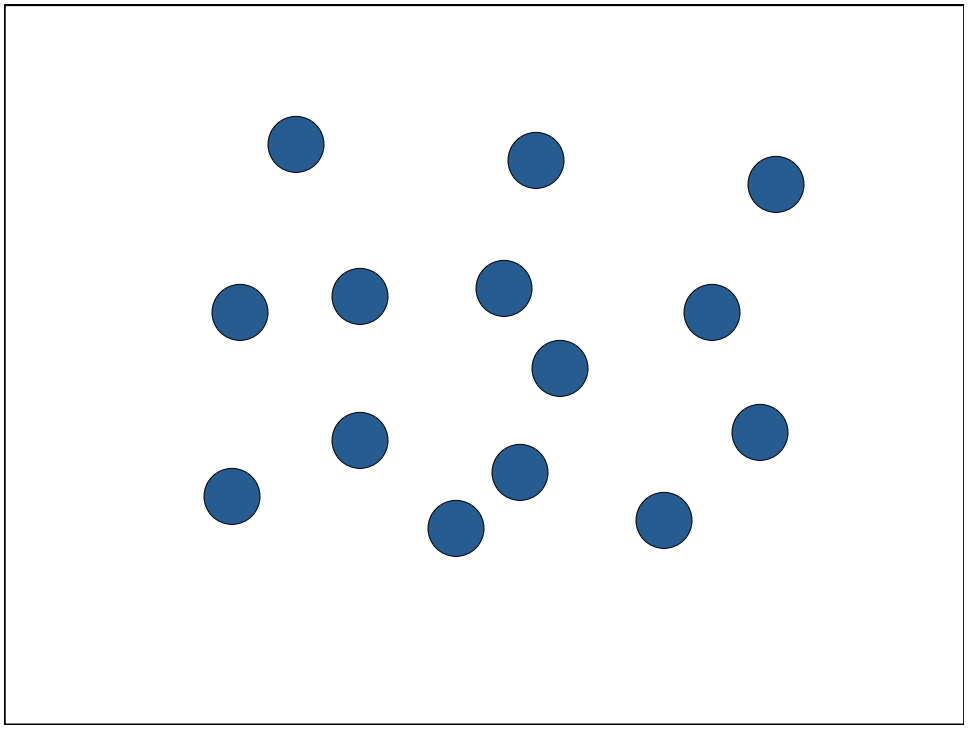
Which has more?





2nd had more or less dots?



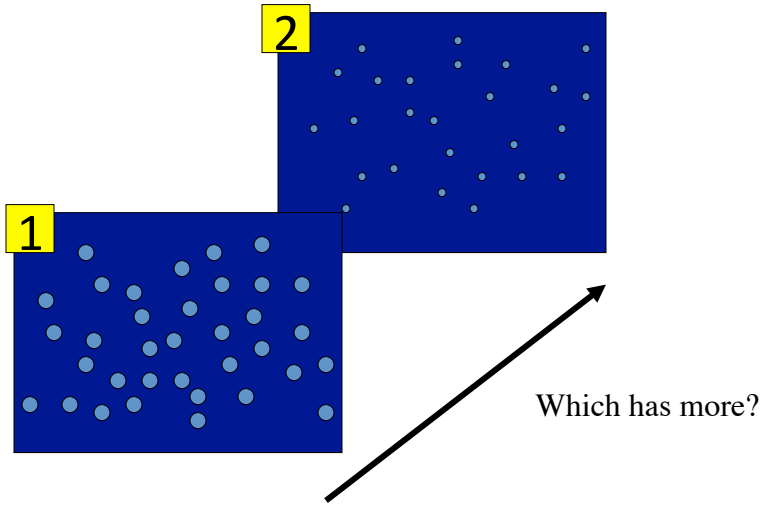


2nd had more or less dots?

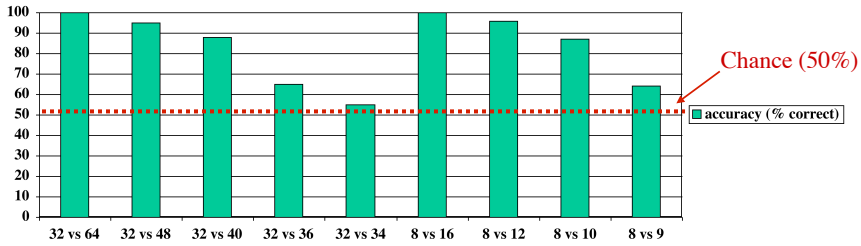
Approximate Number System/Number Sense

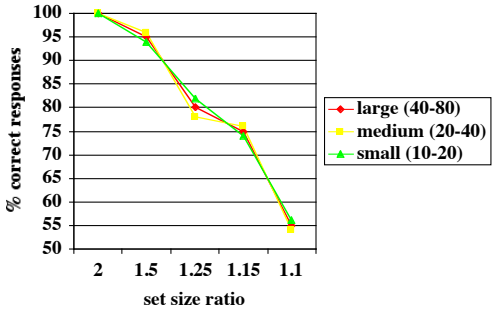
- Allows for rough approximation and comparison of number without counting

Approximate number comparison (Barth, Kanwisher, & Spelke, 2003)



Numerosity Discrimination by Adults





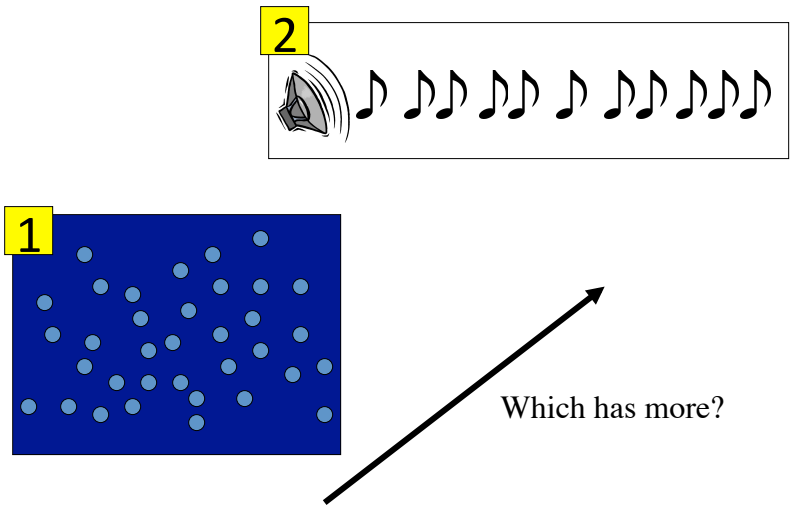
Ratio Signature:
Number discrimination is a function of the ratio between the two numbers to be compared

Number Sense

- Allows for rough approximation (not exact) and comparison of number without counting
- Ratio limited precision

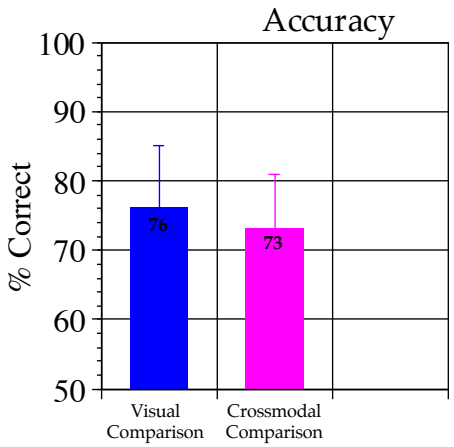
- Comparisons actually based on number or some other property?
  - Size of objects, spacing, density, brightness all controlled over the set

Cross-modal comparisons



Cross-modal results

- Cross-modal comparison is nearly as accurate as visual comparison
- Suggests comparisons are being made over an abstract notion of number rather some direct non-numerical sensory property of the stimulus

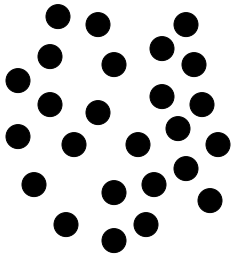


Number Sense

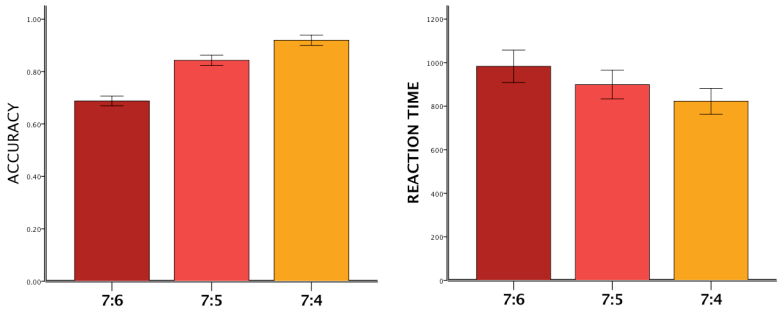
- Allows for rough approximation (not exact) and comparison of number without counting
- Ratio limited precision
- Abstract mental representations/not tied to non-numerical sensory properties

Arithmetic?

LESS

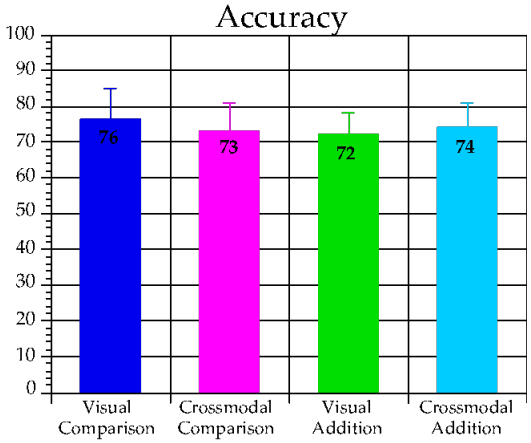


Ratio dependency in arithmetic (between real sum and foil sum)



Hyde & Spelke, in prep

No behavioral cost to cross modal addition

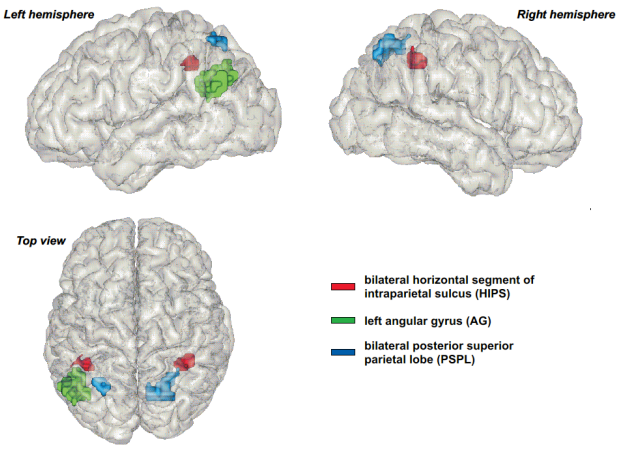


Barth, Kanwisher & Spelke (2003)

Number Sense

- Allows for rough approximation and comparison without counting
- Ratio limited precision
- Abstract mental representations
- Can be used productively for arithmetic

Specialized brain regions for number sense: IPS

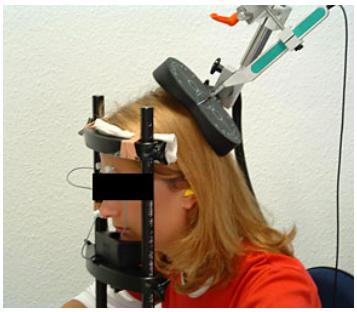


Intraparietal sulcus (IPS)

Dehaene et al., 2003

Impaired IPS = Impaired ANS

- Transcranial magnetic stimulation (TMS) to the IPS will temporarily impair numerical processing (Cappelletti et al., 2009)
- Permanent brain damage to IPS impairs numerical processing



Number Sense

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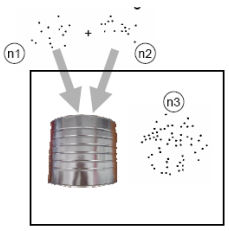
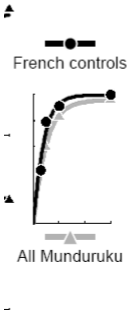
- Can be used productively for arithmetic
- Specialized cortical regions of the parietal lobe

Approximate addition in the Munduruku



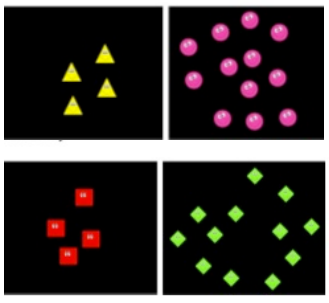


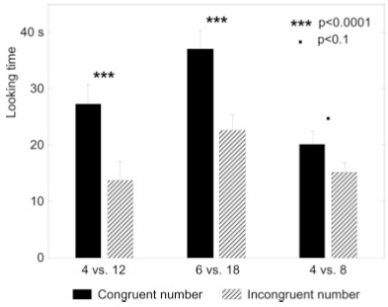
ANS and basic intuitions of arithmetic are universal in humans

(Pica, et al., 2004; Piazza et al., 2011)

Number sense is present from birth

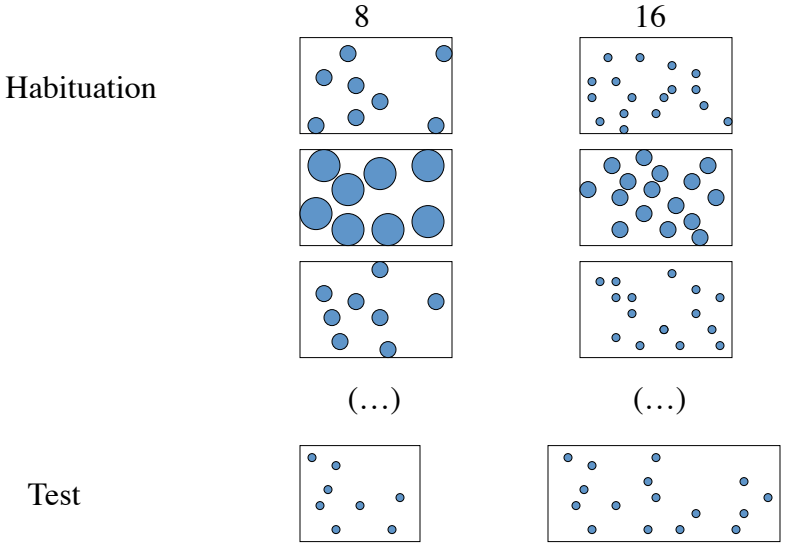
Newborns match number of auditory tones to correct number of visual items (Izard et al., 2009)



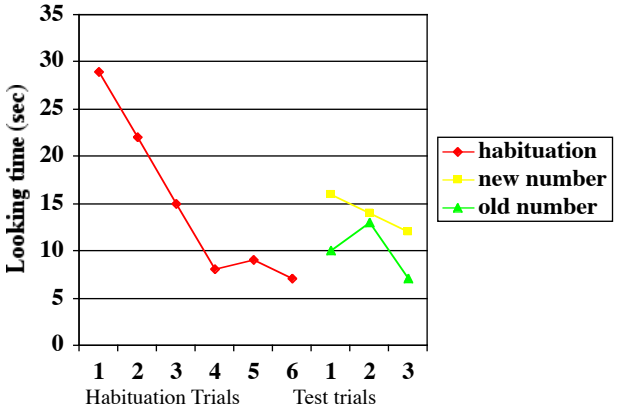


Infant Number Change Detection (Brannon Lab-Duke)

Number sense shows ratio limit in infants



8 vs. 16 success at 6 months



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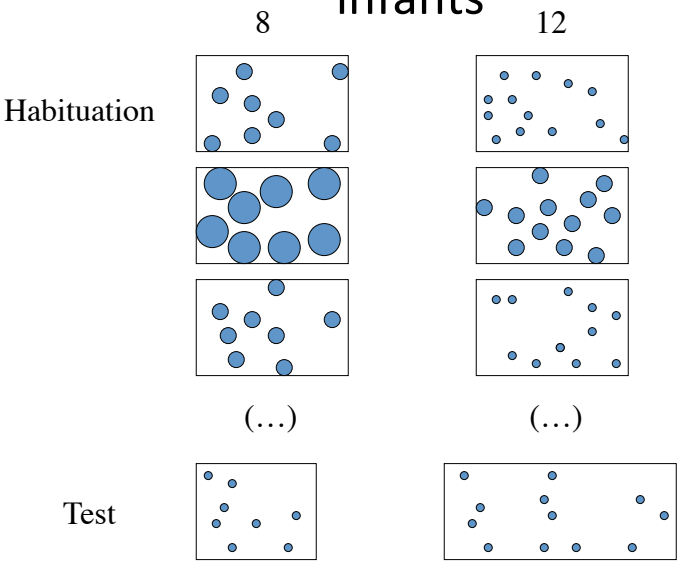
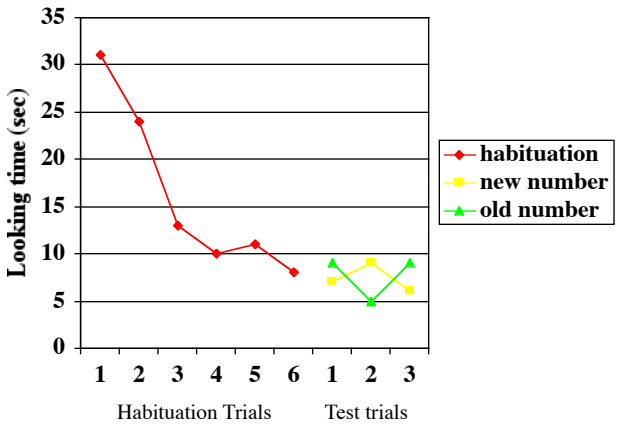


8 vs. 12 failure at 6 months



Further investigations into ratio limits

6 month success	6 month failure
8 vs. 16 dots	8 vs. 12 dots
16 vs. 32 dots	16 vs. 24 dots
4 vs. 8 dots	4 vs. 6 dots

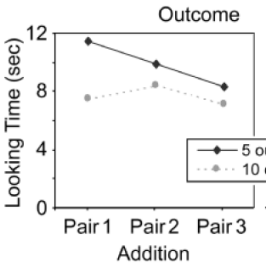
- At 6 months ratio limit is 1:2
- By 9 months this increases in precision to 2:3
- In adults it is ~7:8

Xu, Spelke, Lipton, and others

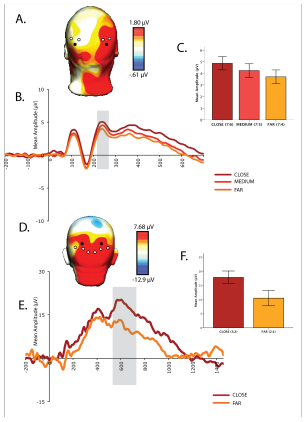
McCrink & Wynn, 2004

Infants can use the number sense to perform addition

McCrink & Wynn, 2004



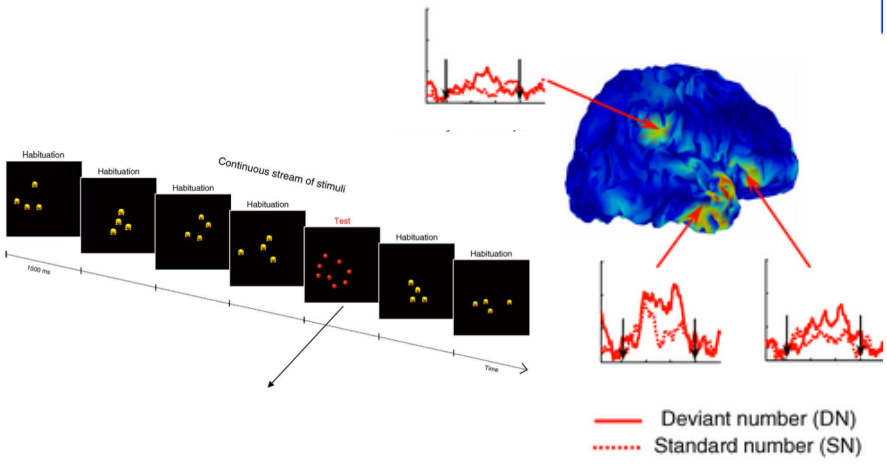
Hyde & Spelke, in prep



Infant Arithmetic

- Infants look longer at unexpected arithmetic outcomes
- Infant brain response scales with the ratio of shown answer to the true answer
- Pattern of activity parallels that seen in adults

Infants engage IPS selectively for number (like adults)

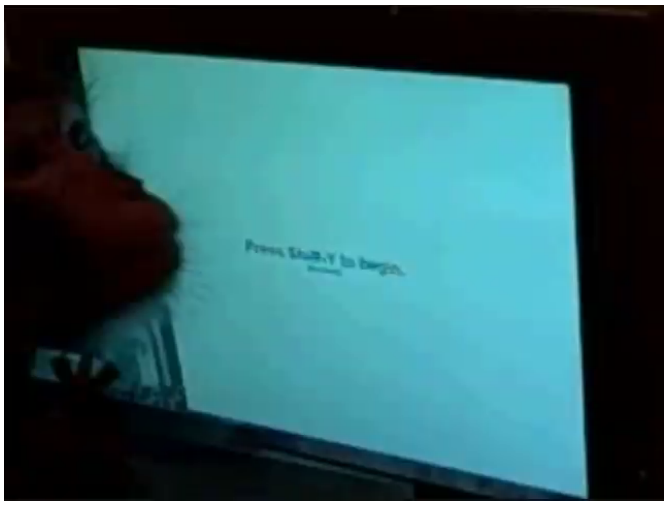


Izard et al., 2008/Hyde et al., 2010

Number Sense

- Allows for rough approximation and comparison without counting
- Ratio limited precision
- Mental representations of number are abstract
- Can be used productively for arithmetic
- Specialized cortical regions of the parietal lobe
- Present from birth, continuous across the lifespan (only changes in precision)

Brannon lab-Duke non-human primate addition and subtraction



Non-human animals have number sense

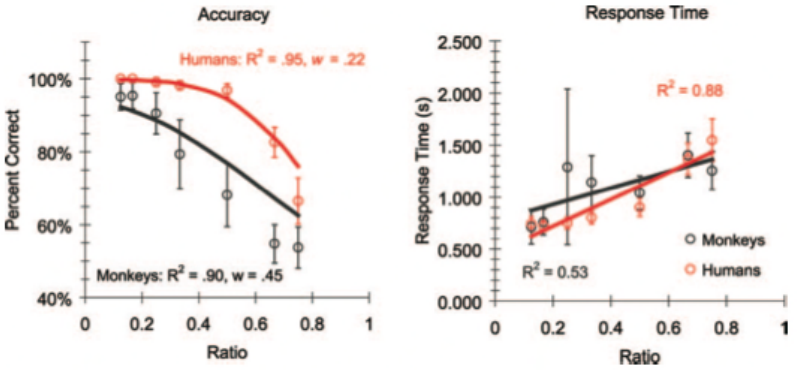
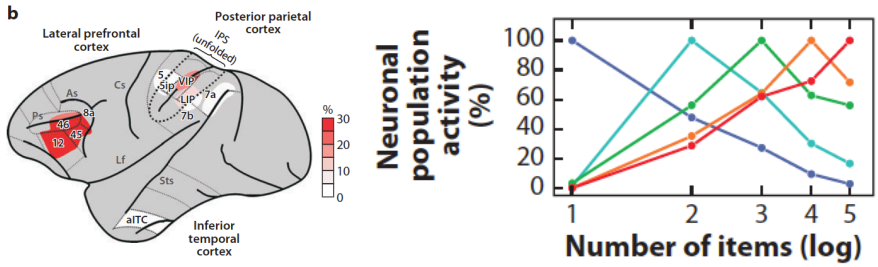


Figure 5. Monkeys Perform Addition like Humans

Animals represent large, approximate numerosities.
Animals' large-number representations show the same signatures as those of human infants & adults.

Single-cell tuning to number in the monkey brain

- Nieder and Dehaene, 2009



Number Sense

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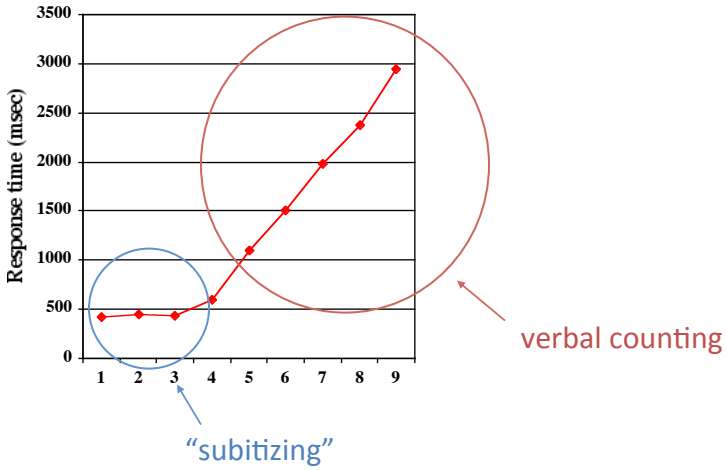
- Ratio limited precision
- Mental representations of number are abstract
- Can be used productively for arithmetic
- Specialized cortical regions of the parietal lobe
- Present from birth, continuous across the lifespan (only changes in precision)
- Shared with many non-human animals

Questions on the ANS/Number Sense?

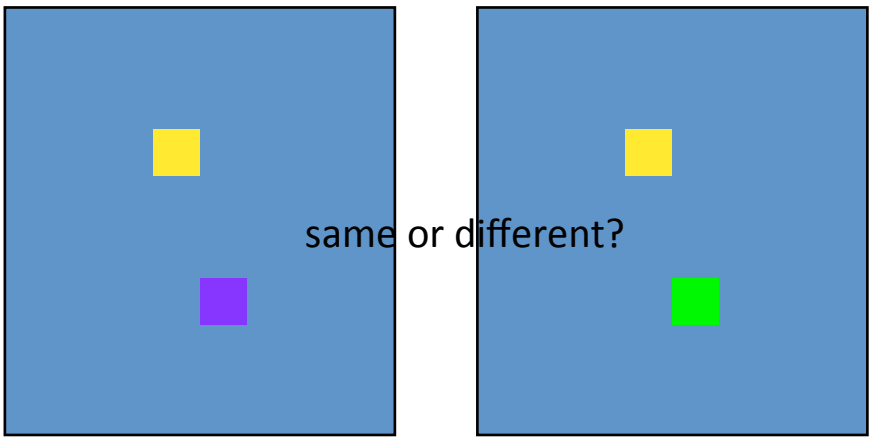
What numerical abilities do we have before passing the conservation task?

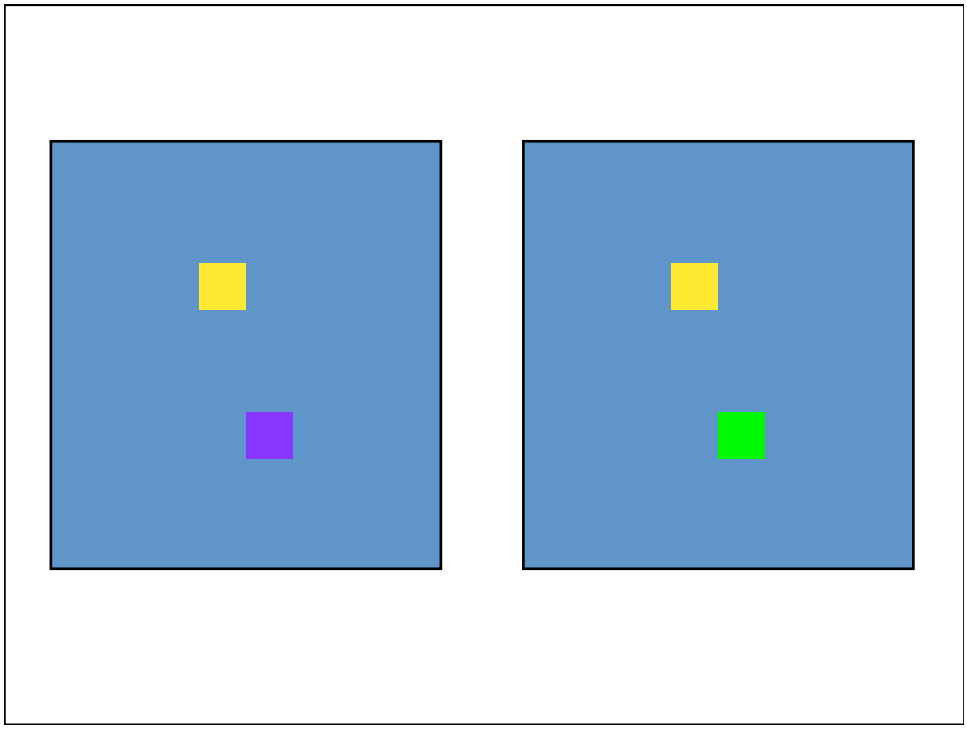
- Two non-verbal, non-symbolic systems
  - Approximate Number System (ANS)
    - "Number Sense"
    - Ability to approximate cardinal value of set
  - Object representation and tracking system (OTS)
    - Ability to represent, remember, distinguish between, and track individual objects

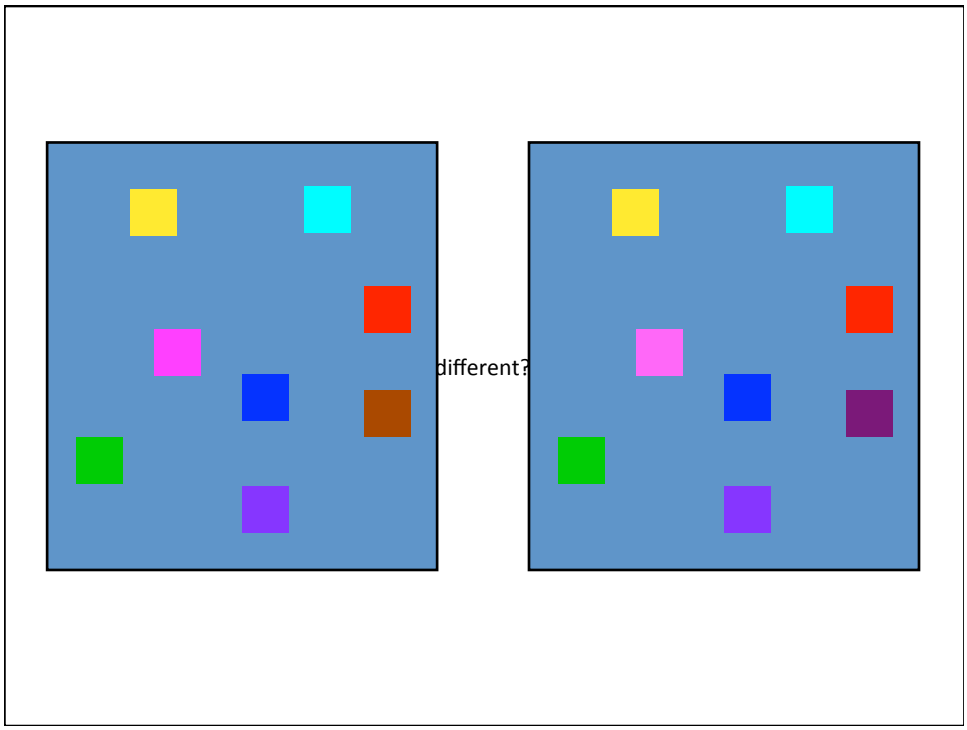


Subitizing (Jevons, 1871)

- Ability to enumerate a limited number of items instantaneously and very accurately
- Usually limited to about 3-4 items in adults



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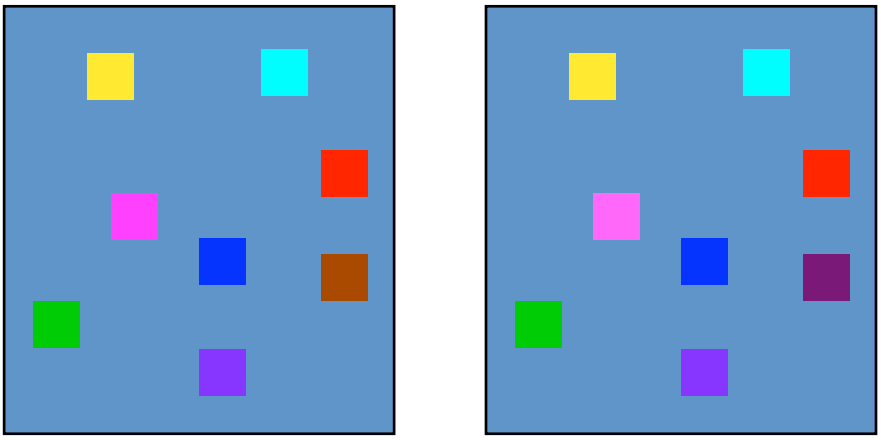
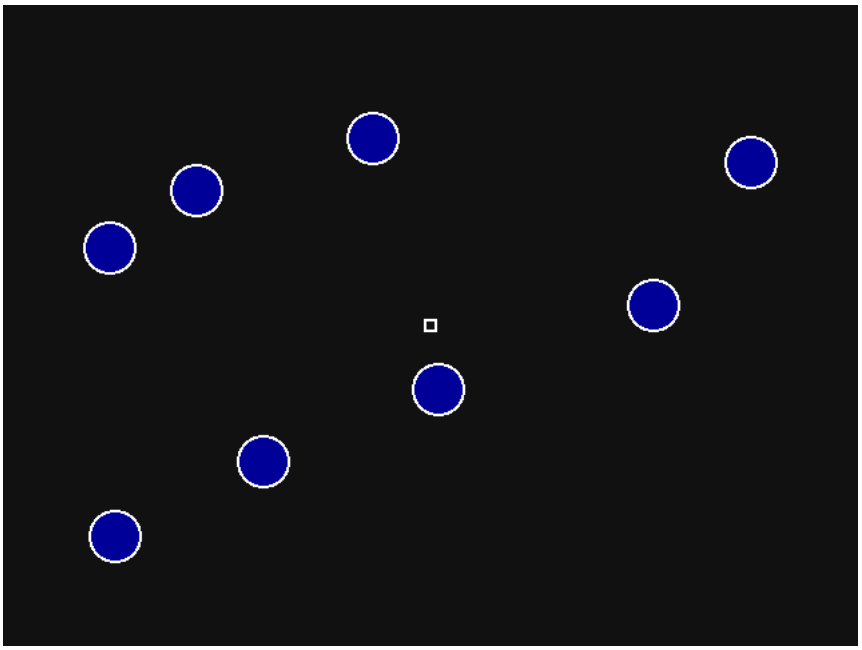


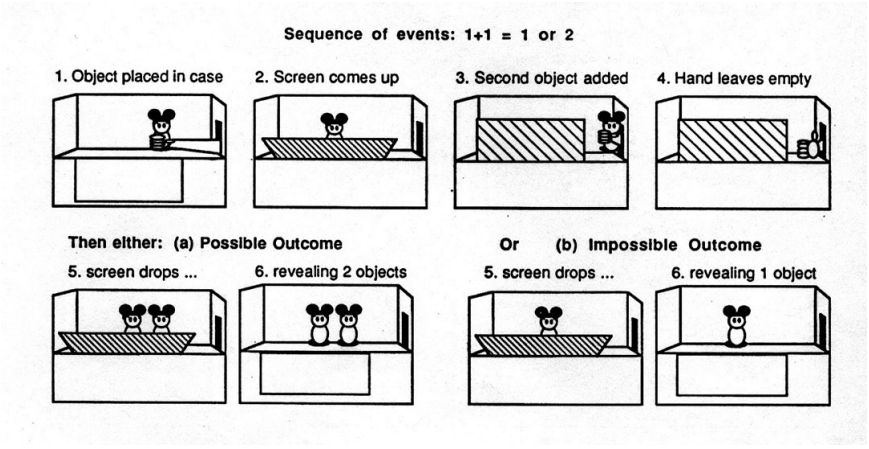
3-4 item capacity limit



Object tracking system (OTS)

- Allows for the simultaneous selection (subitizing), tracking, and remembering of a limited number of individual items
- Comparisons are not ratio dependent
- Limited to about 3-4 total items

Present in infants (Wynn, 1992)



Limitations of OTS in infants (Feigenson et al., 2002)

Succeed at	1 vs. 2
	2 vs. 3
	1 vs. 3
Fail at:	2 vs. 4
	1 vs. 4

Only 3 objects can be tracked in any given location at one time

Questions on the OTS/Parallel Individuation System?

Two Systems of Core Number

(see Feigenson, Dehaene, & Spelke, 2004 reading)

- Two evolutionarily ancient systems that allow for numerical computations
  - Approximate number system (ANS)
    - Allows comparison using approximate numerical magnitudes
  - Object tracking system (OTS)
    - Allows comparison using 1 to 1 correspondence
- Present from early in the development and persist over the lifetime
- Neither has the content to represent the integers/natural number

How does natural number/counting develop?

- Difficult developmental process
- Occurs between 2-5 years of age (in Western societies)
- From learning to recite the count list (one, two, three..) to acquiring integer/natural number concepts usually takes about 2-2.5 years

Is it culturally constructed?

Do core systems play a role in counting development?

Stages of counting development

- Child learn the count list (one, two, three…) but don't understand the meaning of the words
- One-knower: Learn the meaning of "one" (but no other number word meanings)
- Two-knower: Know "one" and learn the meaning of "two" (but no other number word meanings)
- Three-knower: Know "one" and "two" and learn the meaning of "three" (but no other number word meanings)
- Counting principle (CP)-knower: Somewhere around "three" or "four", generalize the counting principles to other numbers.
  - Later-greater rule (the further down the list you go, the larger the number)
  - Successor function (+1 = the next number in the list)

Continuity vs. discontinuity in numerical development

- Core systems do not contain the conceptual information necessary for full understanding of counting/natural number concepts
  - Counting development is not built through simple enrichment of core systems
- Counting development appears to be discontinuous
  - Relies on OTS system and language to make initial associations
  - May rely on ANS for the later/greater insight
  - System that develops is qualitatively different than ANS or OTS or number words alone
  - Allow for exact large cardinal values to be represented and compared.

Universal development?

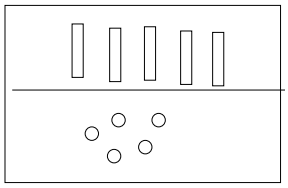
- Does natural number develop in all humans (e.g. object permanence or theory of mind) or is it a cultural invention (e.g. reading/writing)?
- Test cases: Piraha of the Brazilian Amazon (Gordon, 2004; Pica et al., 2004)

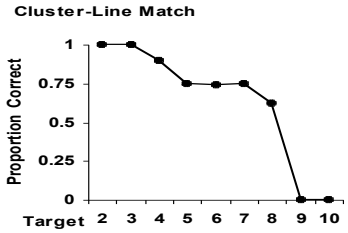
Use of number in Piraha

- Hunter-gatherers in remote region of the Amazon
- No numerical terms beyond “one”, “two”, and “many”
- Peter Gordon (2004) used a variety of matching tasks

Matching Task with Piraha

- Suggests two core systems
  - Perfect performance on small numbers
  - Continually worse performance as number increases for larger numbers
- No ability to match exact large cardinal values (requiring natural number concepts)
- Suggests natural number may not be innate/universal





Associations between symbolic number and core number sense

- Use of the symbolic integer system is influenced by the ANS throughout the lifespan
  - Distance effect when comparing symbolic numbers
  - Activation of common brain regions for symbolic and non-symbolic numbers
- Core number system may be important to development of math
  - Correlation between core number sense and math achievement

Distance Effect

Greater or less than 65?

32

58

96

71

Distance effect

- Closer the comparison the harder it is
  - E.g. 60 vs. 65 is harder than 31 vs. 65
- Signature of the ANS, not the integer system
- Overlap shows up in both behavioral and brain data
  - intraparietal regions respond to both symbolic and non-symbolic comparisons
- Evidence that the ANS is mapped to symbolic numbers

Relationship of number sense to math development

- Basic number sense appears to be related to math development
  - Adult ability to numerically compare arrays of dots is correlated with previous math achievement scores from elementary and high school (Halberda et al., 2008)
  - Ability to do approximate addition is related to math achievement scores in early elementary school children (Gilmore, McCarthy, & Spelke, 2010)

Brief engagement of ANS enhances symbolic math performance

(Khanum, Hyde, & Spelke, in prep)

- Children randomly assigned to practice a few approximate comparison problems or a control task matched for difficulty (e.g. color comparison, length comparison, etc.)
- Children that practiced the approximate number problems perform better on a test of symbolic addition (with up to 3 digits) than children who practiced a variety of other control tasks of equal difficulty

Math development and number sense

- Emerging evidence suggests that symbolic math development can be predicted based on approximate number sense skills.
  - Further research may allow us to identify children early in life that might struggle or excel.
  - Further research may help us understand the relationship of the primitive system to mathematics
- Educators should consider the rich innate numerical and mathematical concepts children bring with them when developing early math and number curriculum

Conclusions

- Humans are born with at least 2 core cognitive systems that give us numerical abilities
- Core number systems help us in learning the symbolic number system of our culture/natural number/integer concepts
- Core number may also be related to learning higher mathematical concepts (e.g. addition).
- Measuring individual differences in core number may allow to intervene to help children develop math skills.